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# OPTIMAL ENVIRONMENTAL STANDARDS UNDER ASYMMETRIC INFORMATION AND IMPERFECT ENFORCEMENT

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# Optimal Environmental Standards Under Asymmetric Information and Imperfect Enforcement*

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**Abstract**

We study optimal policies composed of pollution standards, probabilities of inspection and fines dependant on the degree of noncompliance with the standards, in a context where regulated firms own private information. In contrast with previous literature, we show that optimal policies, being either pooling or separating, can imply violations to strictly positive standards. This result crucially depends on the monitoring costs, the types of firms and the regulator's degree of uncertainty.

**JEL classification:** D82, K32, K42, L51.

**Key words**: standard-setting, costly inspections, convex fines, asymmetric information, noncompliance.

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# 1 Introduction

Very often, environmental regulations require that firms comply with recommended pollution limits or *standards*. However, regulators do not normally have perfect information about the polluting firms, either *ex-ante* or *ex-post*. *Ex-ante* concerns standard setting. Regulators are less informed than firms about their technological characteristics, and they implement mechanisms to elicit private information.[1] *Ex-post* refers to the behavior of firms in response to the standards already in place. Here, regulators do not observe the performance of firms unless they engage in costly monitoring. Therefore, they design *enforcement policies* composed of inspection frequencies and sanctions in case firms are discovered exceeding the standards.[2] Depending on the monitoring costs, the standards to be enforced and the information authorities own about the regulated firms, enforcement may be imperfect, that is, some firms may find it profitable to violate environmental standards.

Recently, Arguedas and Hamoudi (2004) and Arguedas (2004) have studied the characteristics of optimal policies composed of pollution standards, probabilities of inspection and fines, under perfect information *ex-ante* and imperfect enforcement. There, it is shown that optimal policies can induce noncompliance to zero standards only, which is quite an intuitive result. Since fines depend on the degree of noncompliance, if a positive standard that induced noncompliance were set, the regulator would find it profitable to decrease the standard

[1] For example, under the US National Pollutant Discharge Elimination System (NPDES) Program, the Environmental Protection Agency (EPA) issues individual permits to facilities which discharge pollutants into waters of the US, based on reported information about their pollution control processes.

[2] The Civil Penalty Policy of the Clean Water Act establishes the factors that the EPA should consider when imposing sanctions for noncompliance. Among others, the degree of noncompliance is a key gravity factor.

and the probability of inspection at the same time keeping the firm's pollution incentives unchanged and reducing monitoring costs.[3] However, in practise we observe violations of positive standards rather frequently.

In this paper, we show that the *ex-ante* informational constraint plays a key role in the results. We consider a firm that owns private information about its benefits from pollution. The firm can be of two possible types, namely *clean* and *dirty*, based on its induced pollution damages in response to a given policy. The firm is first asked to report that information and, contingent on it, the regulator then sets the optimal policy considering the firm's pollution level in response to the policy. By the *revelation principle*, we can restrict attention to direct mechanisms which induce the firm to reveal its true type. That is, we can concentrate on the subset of *incentive-compatible* policies.[4]

The message of the paper is clear. Under ex-ante imperfect information, we can find situations where it is optimal to set positive standards that induce noncompliance. The result is independent of the optimal policy being pooling (the same policy for both types) or separating (contingent on type).

In the case of a pooling policy, we show that policies that induce full compliance are never optimal, since it is always worth to infinitesimally decrease the probability of inspection: the savings in monitoring costs are larger than the decrease in welfare associated with both types' larger pollution levels. As we have already pointed out, a positive standard is never optimal in the com-

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[3] This is similar to Becker (1968)'s well known result of imposing maximal fines to keep enforcement costs at the minimum. Given a pollution level and a structure of fines dependant on the degree of noncompliance, a lower standard increases the fine for noncompliance and, therefore, it is possible to decrease the probability of inspection, then saving monitoring costs.

[4] Our approach differs from that in which, given the standard, the firm reports its emission level with the possibility of under-reporting, such as in Sandmo (2002). In our case, we have an added *ex-ante* informational asymmetry and, since we analyze optimality of the standards, we can restrict ourselves to incentive compatible policies. Also, once emissions have been released, we assume that they can be measured through costly monitoring.

plete information case. However, a zero standard under incomplete information may result in over-enforcement of the clean type, with the corresponding negative effect on social welfare. This result is relevant and provides an additional explanation to the literature in favor of non-maximal fines.[5]

In fact, we find violations to strictly positive standards under low monitoring costs, intermediate values of the clean type's profitability and large regulator's uncertainty. The explanation is very intuitive, since there exists a trade-off between enforcement costs and the clean type's over-enforcement problem. Given clean type's profitability, the larger the monitoring costs, the larger the enforcement costs, which favors a zero standard setting. By contrast, given a level of the monitoring costs, the smaller clean type's profitability, the larger the over-enforcement problem, which favors a positive standard setting. However, for a sufficiently low clean type's profitability, a positive standard may not be possible, since the full noncompliance region in this case may be very small. Finally, when uncertainty decreases (in favor of any type) , the solution approximates to the complete information solution, which implies a zero standard.

By contrast, if the policy is separating, both the standard and the probability of inspection are smaller for the dirty type, to preserve incentive compatibility.[6] Also, the dirty type always finds it profitable to violate the standard, which again can be positive under low monitoring costs. However, as opposed to the pooling case, we now find that it is more likely to find this result when both clean type's profitability and likelihood are large, since in these two cases,

[5] After Becker (1968), several papers in the crime context have explained the reasons why fines are not maximal, such as risk aversion (Polinsky and Shavell (1979)), imperfect information about the regulatory policy (Bebchuk and Kaplow (1991), Kaplow (1990)), differences in wealth (Polinsky and Shavell (1991)) or marginal deterrence (Andreoni (1991), Shavell (1992), Heyes (1996)). In all these papers, however, standards are exogenous.

[6] In the tax evasion literature, the optimal inspection probability is also a decreasing function of reported income. For example, see Reinganum and Wilde (1985).

the solution approximates to the clean type's compliance solution. Here, the trade-off is between enforcement costs and dirty type's under-enforcement. The larger the clean type's profitability and likelihood, the larger the dirty type's under-enforcement problem if its standard is zero.

The literature on standards and enforcement issues started with Downing and Watson (1974) and it is vast nowadays (Heyes (2000) provides a comprehensive survey in the environmental context). However, our approach has not been considered yet, namely combining standard-setting, endogenous imperfect enforcement and asymmetric information. This allows us to rationalize positive standard violations, a result that is not possible under alternative assumptions within the principal-agent framework. For example, Ellis (1992a) and Gottinger (2001) study standard-setting under *ex-ante* incomplete information, but they restrict attention to policies which induce compliance. There are papers which study incentive compatible optimal pollution taxes, such as Jebjerg and Lando (1997), which implicitly constrain to zero standards. Swierzsbinky (1994) consider optimal taxation also, relaxing the assumption of incentive compatibility, but they again restrict to zero standards. The only exception is Arguedas (2005), which considers a bargaining context under complete information and assumes that the firm can choose the environmental technology as well. There, it may be beneficial for both the regulator and the firm to achieve a cooperative agreement where the firm chooses a cleaner technology in exchange for a relaxed regulation consisting of a positive standard and a reduced fine for noncompliance.

The remainder of the paper is organized as follows. In the next section, we present the model. In Section 3, we study the optimal behavior of the firm. In Section 4, we analyze the characteristics of the optimal pooling policy and the likelihood of obtaining positive standards. In Section 5, we discuss the case of the optimal separating policy. We conclude in Section 6. All the proofs are in the Appendix.

## 2 The Model

We consider a firm that generates pollution as a by-product of its production activity. The firm obtains private profits which depend on the pollution level $e \geq 0$ and a parameter $\theta_i > 0$, $i = 1, 2$, $\theta_1 < \theta_2$, which refers to the firm's pollution profitability. Let $B(e, \theta_i) = \theta_i b(e)$ represent the firm's profits, where $b(e)$ is continuous and concave in the pollution level with an interior maximum at $\widetilde{e} > 0$, and such that $b(0) = 0$ and $b'''(e) \geq 0$.[7] Given a pollution level, the clean type $(\theta_1)$ obtains lower pollution profits and marginal profits than the dirty type $(\theta_2)$. The firm knows its type but the regulator only knows the probability distribution of the types. Let $\gamma_i$ denote the probability that the firm is of type $\theta_i$, such that $\gamma_i \in [0, 1]$ and $\gamma_1 + \gamma_2 = 1$.

Pollution generates external damages measured by $d(e)$, which is continuous, strictly increasing and convex in the pollution level, and such that $d(0) = 0$.

In the absence of regulation, the firm does not internalize external damages and pollution is $\widetilde{e} = \arg\max_{e \geq 0} \theta_i b(e)$, for all $i$.

We assume there exists a regulator who sets a standard $s \in [0, \widetilde{e}]$, that is, a maximum level of permitted pollution.[8] The regulator cannot observe the pollution level unless it monitors the firm, which is costly and perfectly accurate.

[7] This specification of profits simplifies the algebra without affecting the qualitative nature of the results.

[8] Obviously, the regulator is not interested in a standard larger than the pollution level chosen by the firm in the absence of regulation.

The cost *per* inspection is $c > 0$. Therefore, the regulator does not generally inspect the firm in every instance but only with probability $p \in [0,1]$. Once inspected, if the firm is discovered exceeding the standard, then it is forced to pay a penalty which depends on the degree of noncompliance, $e-s$. We assume that the sanction is represented by the function $F(e-s)$, which is quadratic, strictly increasing and convex in $e-s \geq 0$, and such that $F(e-s) = 0$ for all $e-s \leq 0$. Given these assumptions, we have that $(F')^2 - FF'' > 0$ for all $e-s \geq 0$, a property that plays a key role in the results, as we will see later on. We assume that the sanction is fixed by a government entity other than the regulator, for example, the judiciary.[9]

We consider a principal-agent framework where the regulator (principal) chooses the standard and the probability of inspection that maximizes social welfare, considering the optimal response of the firm (agent) to the policy.

Given $\{s,p\}$, a firm of type $\theta_i$ chooses the pollution level that maximizes its expected payoff, that is, private profits minus expected penalties, as follows:

$$P(s,p,\theta_i) = \max_{e \geq 0} \{\theta_i b(e) - pF(e-s)\} \tag{1}$$

Let $e(s,p,\theta_i)$ be the pollution level chosen by type $\theta_i$ given the policy $\{s,p\}$, i.e., $e(s,p,\theta_i) = \arg\max_{e \geq 0} \{\theta_i b(e) - pF(e-s)\} \leq \widetilde{e}$.

Considering the firm's best response, the regulator now chooses the policy

[9] This assumption is common in the literature in this context except, for example, in Heyes (1996) or Arguedas (2005). In other contexts, such as crime, there are several papers which determine optimal fines and inspection probabilities, such as Becker (1968), Polinsky and Shavell (1979, 1990) or Bebchuck and Kaplow (1991), but there the standard is exogenous. In the context of tax evasion, few papers consider endogenous fines. See, for instance, Mookherjee and Png (1989).

that maximizes social welfare. Since the regulator does not know the true type of the firm, the policy cannot be based upon it. There are two kind of policies the regulator may choose. The first is a *pooling (or uniform) policy* $\{s, p\}$, that is, the same policy regardless of the type. In this case, the regulator does not need to elicit any information from the firm and social welfare is as follows:

$$SW(s,p) = \sum_{i=1}^{2} \gamma_i \left[ P(s,p,\theta_i) - d(e(s,p,\theta_i)) + pF(e(s,p,\theta_i) - s) \right] - cp \quad (2)$$

The regulator is concerned about the firm's expected payoff, the generated damages, the expected collected fines and the expected monitoring costs. We assume that there are no social costs associated with collecting fines, and that fines are redistributed lump-sum. Also, we do not impose any budget requirement on the monitoring activity. Considering (1), (2) reduces to:

$$SW(s,p) = \sum_{i=1}^{2} \gamma_i \left[ \theta_i b(e(s,p,\theta_i)) - d(e(s,p,\theta_i)) \right] - cp \quad (3)$$

The second type of policy is *separating*, that is, a policy contingent on type. Here, the regulator has to design a mechanism to elicit the firm's private information. By the *revelation principle*, we can concentrate on direct mechanisms where the regulator asks the firm to report its type, $\widehat{\theta}_i$, and then, it sets the policy based on the report, $\left\{ s\left(\widehat{\theta}_i\right), p\left(\widehat{\theta}_i\right) \right\}$, such that it induces the firm to reveal its true type, $\widehat{\theta}_i = \theta_i$. This is the well known *incentive compatibility condition*, represented as follows:

$$\theta_i \in \arg\max P\left( s\left(\widehat{\theta}_i\right), p\left(\widehat{\theta}_i\right), \theta_i \right) \quad (4)$$

For convenience, we assume that if the firm is indifferent between announcing any of the two types, then it announces the true type.

Denoting $s_i = s\left(\widehat{\theta}_i\right)$ and $p_i = p\left(\widehat{\theta}_i\right)$, $i = 1, 2$, social welfare is now:

$$SW(s_1, s_2, p_1, p_2) = \sum_{i=1}^{2} \gamma_i \left[\theta_i b\left(e\left(s_i, p_i, \theta_i\right)\right) - d\left(e\left(s_i, p_i, \theta_i\right)\right) - cp_i\right] \qquad (5)$$

where $(s_1, s_2, p_1, p_2)$ satisfy (4). Observe that a uniform policy is trivially incentive compatible.[10]

An important assumption of our model is that the regulator commits to the announced inspection probability. This assumption can be justified considering that the regulator has to build up a reputation, that is, policy announcements must be credible to induce the desired behavior.[11]

In the next section, we study the firm's induced behavior with respect to the announced policy.

# 3 The Optimal Behavior of the Firm

Consider a feasible policy $\{s, p\}$. As explained in the previous section, the corresponding type $\theta_i$'s expected payoff is given by $(1)$.

If type $\theta_i$ complies with the standard $(e < s)$, it does not incur any penalty. Since $b(e)$ is strictly increasing in $e \leq \widetilde{e}$, the *optimal compliance decision* is $s$ and its payoff is $\theta_i b(s)$.

[10] Besides incentive compatibility, the literature on economics of information considers participation constraints also, that is, feasible policies must be such that firms' payoffs are non-negative. In our case, this additional requirement is trivially satisfied since $b(0) = 0$.

[11] A formal justification of this assumption would require to consider a dynamic model, which is beyond the scope of this paper. In static models such as ours, the assumption of commitment is common in the literature. Some exceptions in the environmental context are Ellis (1992b), Grieson and Singh (1990) or Franckx (2002).

If type $\theta_i$ exceeds the standard ($e > s$), then there is a chance that is inspected and punished. Consequently, the *optimal noncompliance decision* is $n(s,p,\theta_i) = \arg\max_{e>s} \{\theta_i b(e) - pF(e-s)\} > s$ and the corresponding payoff is $\pi(s,p,\theta_i)$. Since the maximand is strictly concave in $e$, the first order condition characterizes the interior noncompliance decision:

$$\theta_i b'(e) = pF'(e-s) \tag{6}$$

Implicitly differentiating (6), we obtain $n_{ip} = n_p(s,p,\theta_i) = \frac{F'}{\theta_i b'' - pF''}$ and $n_{is} = n_s(s,p,\theta_i) = -\frac{pF''}{\theta_i b'' - pF''}$. Observe that $n_{ip} < 0$ and $0 \leq n_{is} < 1$. That is, type $\theta_i$'s pollution level increases when the probability of inspection decreases and the standard increases. However, since $n_{is} < 1$, the degree of violation decreases when the standard increases.[12]

Given $\{s,p\}$, type $\theta_i$ chooses whether to comply or not depending on the expected payoff of each possibility. Thus, its optimal response is:

$$e(s,p,\theta_i) = \begin{cases} s & \text{if } \theta_i b(s) \geq \pi(s,p,\theta_i) \\ n(s,p,\theta_i) & \text{if } \theta_i b(s) < \pi(s,p,\theta_i) \end{cases} \tag{7}$$

and its expected payoff can be further expressed as:

$$P(s,p,\theta_i) = \max\ \{\theta_i b(s), \pi(s,p,\theta_i)\} \tag{8}$$

In the following lemma, we show the properties of the function $P(s,p,\theta_i)$:

**Lemma 1** *The function $P(s,p,\theta_i)$ is non-decreasing and concave in $s$, non-*

[12] Note that $n_{is} = 0$ when either $F'' = 0$ or $p = 0$.

*increasing and convex in $p$, it has a nonnegative cross partial, and it is such that $P(s,p,\theta_2) > P(s,p,\theta_1)$. Moreover, $P(s,0,\theta_i) = \theta_i b(\widetilde{e})$ for all $i$.*

We now characterize the set of policies for which each type is indifferent between complying and noncomplying with the standard. Since sanctions are continuous at $e = s$, the maximand in (1) is continuous for all $s$. Therefore, considering (6), type $\theta_i$ complies with the standard only if $\theta_i b'(s) \leq pF'(0)$. Thus, the minimum probability that induces type $\theta_i$ to comply with $s$ is:

$$p^c(s,\theta_i) = \frac{\theta_i b'(s)}{F'(0)} \tag{9}$$

which is decreasing and convex in $s$, and such that $p^c(s,\theta_2) > p^c(s,\theta_1)$.[13] Since $p \leq 1$, there may exist a subset of nonenforceable standards for each $\theta_i$.[14]

In Figure 1, we represent the functions $p^c(s,\theta_i)$ in the space of feasible policies. In the horizontal axis we measure the standard and in the vertical axis, we measure the probability of inspection. These functions divide the set of feasible policies into three regions, namely *compliance* (C), *partial compliance* (PC) and *noncompliance (NC)*. Therefore, all the policies on or above the function $p^c(s,\theta_2)$ induce both types to comply with the standard. The set of policies between $p^c(s,\theta_1)$ and $p^c(s,\theta_2)$ induce the clean type to comply only. Finally, the policies below the function $p^c(s,\theta_1)$ induce both types to violate the standard. Thus, $\theta_2$'s noncompliance region is larger than that of $\theta_1$.

In the figure, we also include each type's indifference map, where each indifference curve is composed of the set of policies such that type $\theta_i$´s expected payoff is constant. By Lemma 1, type $\theta_i$'s payoff increases to the southeast,

[13] The assumptions on the penalty function ensure that $F'(0)$ is finite and strictly positive.

[14] If there exists $\widehat{s}_i > 0$ such that $p^c(\widehat{s}_i,\theta_i) = 1$, then $s \in [0,\widehat{s}_i)$ cannot be enforced for $\theta_i$.

i.e., whenever the standard is larger and the probability of inspection is smaller. And it obtains the maximum expected payoff at $s = \widetilde{e}$, $p \in [0,1]$ and $s \geq 0$, $p = 0$. The shape of the indifference curves is now presented in the following:

**Lemma 2** *If a policy $\{s, p\}$ induces type $\theta_i$ to comply with the standard, the indifference curve at that policy is vertical. If it induces noncompliance, the indifference curve at that policy is strictly increasing and convex. At any $\{s, p\}$, the slope of $\theta_1$'s indifference curve is no smaller than that of $\theta_2$.*

In both the full noncompliance and the partial compliance regions, indifference curves satisfy the *single crossing property.* However, in the full compliance region, indifference curves do not cross.

The revelation principle allows us to restrict attention to incentive compatible policies. For example, a policy $\{s_1, p_1\}$ for $\widehat{\theta_1}$ and a policy $\{s_2, p_2\}$ in the shaded area of Figure 1 for $\widehat{\theta_2}$ is incentive compatible, i.e., no type has an incentive to misrepresent its type. Note that $s_2 \leq s_1$ and $p_2 \leq p_1$.

Having studied the firm's optimal response, we now analyze the features of the optimal policy. First, we consider the case of the uniform policy.

# 4 The Optimal Pooling Policy

In this section, we analyze the case in which the regulator sets the same policy regardless of the type. Here, the regulator maximizes social welfare given by (2), considering the firm's optimal behavior analyzed in the previous section. In the following proposition, we provide a useful preliminary result to characterize the optimal policy in this case.

**Proposition 3** *Let $(s^*, p^*)$ be the optimal pooling policy. Then, $p^* \leq p^c(s^*, \theta_1)$.*

Therefore, a pooling policy which induces full compliance is never optimal, since, by (9), $p^c(s^*, \theta_1) < p^c(s^*, \theta_2)$. A policy as such would imply clean type's under-enforcement and dirty type's over-enforcement with respect to the complete information case (see Figure 2). Intuitively, full compliance is socially too expensive, and welfare increases if the regulator decreases the inspection probability, since clean type's incentives remain unchanged, and the savings in monitoring costs are larger than the decrease in efficiency due to the larger dirty type's induced pollution level.

Thus, if the optimal policy is pooling, at least it induces the dirty type to violate the standard. The clean type cannot strictly prefer to comply with the standard at the optimal policy. If a policy as such were set, welfare could be increased decreasing the probability of inspection, since incentives for the clean type would remain unchanged and we would overcome the dirty type's over-enforcement problem.

Consequently, the optimal pooling policy is obtained as follows:

$$\begin{aligned} & Max_{s,p} \quad \sum_{i=1}^{2} \gamma_i \left(\theta_i b\left(e_i\right) - d\left(e_i\right)\right) - cp \\ s.t. \quad & p \leq p^c\left(s, \theta_1\right) \\ & s \geq 0 \end{aligned} \tag{10}$$

where $e_i = e(s, p, \theta_i)$ is given by (7).[15] The optimality conditions are summarized in the following:

[15] For the sake of clarity, we assume that the optimal probability of inspection is included in the interval $[0, 1]$. This is also valid in Proposition 7.

**Proposition 4** *The optimal pooling policy* $(s^*, p^*)$ *is such that:*

$$\sum_{i=1}^{2} \gamma_i \left(\theta_i b'(n_i) - d'(n_i)\right) n_{ip} - c - \lambda = 0 \tag{11}$$

$$\sum_{i=1}^{2} \gamma_i \left(\theta_i b'(n_i) - d'(n_i)\right) n_{is} + \lambda \frac{\partial p^c}{\partial s} + \eta = 0 \tag{12}$$

$$\lambda \geq 0,\ p^* \leq p^c(s^*, \theta_1),\ \lambda\left(p^* - p^c(s^*, \theta_1)\right) = 0$$

$$s^* \geq 0,\ \eta \geq 0,\ \eta s^* = 0$$

*where* $(\lambda, \eta)$ *are the Lagrange multipliers associated with problem (10) and* $n_i$ *is type* $\theta_i$*'s optimal response to* $(s^*, p^*)$ *given by* $(6)$.

Figures 3 and 4 represent the cases of partial compliance and full noncompliance, respectively, both of them compatible with the solution. In the figures, we have included the social welfare contours, where each contour represents the set of policies $(s, p)$ such that social welfare remains constant.

We cannot generally conclude that the optimal standard is zero in any case. If $p^* = p^c(s^*, \theta_1)$, the case of Figure 3, the optimal policy induces partial compliance and the optimal standard is generally positive (the contrary would require to enforce type $\theta_1$ to comply with a zero standard, see footnote 14). Therefore, the dirty type violates a positive standard, a result that is not possible under complete information. In this case, combining (11) and (12), we obtain that the optimal standard and inspection probability are such that the marginal rate of substitution in terms of optimality of the induced pollution levels must equal the marginal rate of substitution to ensure type $\theta_1$'s compliance.

If $p^* < p^c(s^*, \theta_1)$, the case of Figure 4, the optimal policy induces full noncompliance. Here, the optimal standard need not be zero either, as opposed to the one type case.[16] Therefore, it is possible that both types violate positive standards. Since $\lambda = 0$, even a positive standard implies that $\theta_1 b'(n_1) - d'(n_1) > 0$, which means that type $\theta_1$ is over-enforced.[17] Therefore, a zero standard could restrict type $\theta_1$'s pollution even more, with the corresponding welfare decrease. By contrast, type $\theta_2$ is under-enforced.

In general, we can conclude that the most likely solution is that of Figure 4, except when monitoring costs are small or when the full noncompliance region is small (or equivalently, when type $\theta_1$'s profitability is small). By (11), it is easy to see that the monitoring costs and the optimal inspection probability are negatively related. Thus, the smaller the monitoring costs, the larger the inspection probability and, therefore, the more likely that the solution induces partial compliance. Also, the smaller the full noncompliance region, the larger the likelihood that the inspection probability induces partial compliance. The following corollary shows that, regardless of $\theta_1$, the optimal pooling policy induces partial compliance in the extreme case where $c = 0$.

**Corollary 5** *If $c = 0$, the optimal pooling policy induces partial compliance.*

When the optimal policy induces full noncompliance, the standard is positive for some values of the parameters. As we have already pointed out, type $\theta_1$ is over-enforced at the solution. On one hand, the smaller the standard, the larger the over-enforcement problem. On the other hand, the smaller the standard, the smaller the enforcement costs of inducing a particular pollution level.

[16] In the case of a unique type $\theta_i$, by (11), we can easily see that $\theta_i b'(n_i) - d'(n_i) < 0$, since $n_{ip} < 0$, $c > 0$ and $\lambda = 0$. But then, $\eta > 0$, by (12), since $n_{is} > 0$, which implies that $s^* = 0$.

[17] If $s^* > 0$, then $\eta = 0$. By (12), we have $\gamma_1 A_1 n_{1s} = -\gamma_2 A_2 n_{2s}$, where $A_i = \theta_i b'(n_i) - d'(n_i)$, which implies that $c = \gamma_2 A_2 \left(n_{2p} - \frac{n_{2s}}{n_{1s}} n_{1p}\right)$. Since $n_{2p} - \frac{n_{2s}}{n_{1s}} n_{1p} = \frac{F'(n_2 - s) - F'(n_1 - s)}{\theta_2 b'' - pF''} < 0$ and $c > 0$, we then have $A_2 < 0$ and $A_1 > 0$.

Therefore, there exists a trade-off between the over-enforcement problem and the enforcement costs. Thus, the larger (smaller) the monitoring costs, the more likely the optimal standard is zero (positive). By contrast, the larger (smaller) type $\theta_1$'s profitability, the less (more) important the over-enforcement problem and the more likely the optimal standard is zero (positive). In consequence, the larger $\theta_1$, the smaller the interval of the monitoring costs for which the standard is positive. Finally, a positive standard is more likely under large uncertainty, that is, when $\gamma_1$ takes intermediate values. This is so because under low uncertainty, the solution approximates to the complete information solution, where the optimal standard is zero (see Figure 4).

The following example illustrates all these features.

### 4.1 Example 1

Consider the specific functional forms:

$$b(e) = \begin{cases} e, & e \leq 1 \\ 1 - e, & e > 1 \end{cases}$$

$$d(e) = e^2$$

$$F(e - s) = (e - s) + (e - s)^2$$

Observe that $\widetilde{e} = 1$.[18] For simplicity, we fix $\theta_2 = 1$. Therefore, $\theta_1 < 1$.

[18] Note that $b(e)$ is linear, which considerably simplifies the algebra without affecting the results. Here, $p^c(s, \theta_i) = \frac{\theta_i}{F'(0)}$, i.e., $\theta_i$'s threshold probability does not depend on $s$.

We compute the optimal pooling policy applying Proposition 4:

$$\gamma_1\theta_1\frac{p(\theta_1+1-2s)-\theta_1}{2p^3}+(1-\gamma_1)\frac{2p(1-s)-1}{2p^3}+c+\lambda=0$$
$$\gamma_1\frac{p(\theta_1+1-2s)-\theta_1}{p}+(1-\gamma_1)\frac{2p(1-s)-1}{p}+\eta=0$$
$$\lambda(p-\theta_1)=0,\ \lambda\geq 0,\ p\leq\theta_1$$
$$\eta s=0,\ s\geq 0,\ \eta\geq 0$$

We now explore the likelihood of obtaining optimal positive standards. Figure 5 illustrates the relationship between the optimal standards and the monitoring costs for different values of $\theta_1$, in the case of large uncertainty ($\gamma_1 = 0.5$). If $\theta_1 = 0.8$ for example, the optimal pooling solution induces partial compliance to $s = 0.3875$ if $c \in [0, 0.002]$. If $c \in [0.002, 0.0024]$, the optimal pooling policy induces full noncompliance to a positive standard, which decreases when the monitoring cost increase. Finally, if $c > 0.0024$, the optimal standard is zero. If $\theta_1$ is lower, the solution induces partial compliance for a larger interval of the monitoring costs. This is intuitive since the lower $\theta_1$, the lower the full noncompliance region, and therefore, the larger the restriction for the full noncompliance solution to exist. If $\theta_1 = 0.5$, we now obtain a larger interval of the monitoring costs for which the optimal standard is positive, $c \in [0.125, 0.227]$. For $\theta_1$ sufficiently small, we do not obtain full noncompliance to positive standards.

A similar picture can be obtained under alternative values of $\gamma_1$, that is, under different degrees of uncertainty. For example, if $\gamma_1 = 0.1$, the range of monitoring costs for which we obtain full noncompliance to a strictly positive standard is $c \in [0.0007, 0.007]$ if $\theta_1 = 0.8$ and $c \in [0.045, 0.049]$ if $\theta_1 = 0.5$.

Alternatively, if $\gamma_1 = 0.9$, we have $c \in [0.0007, 0.01]$ and $c \in [0.045, 0.16]$ for $\theta_1 = 0.8$ and $\theta_1 = 0.5$, respectively. In these two cases, uncertainty has decreased with respect to the case in which $\gamma_1 = 0.5$ and, consequently, the interval of the monitoring costs for which we obtain violations to strictly positive standards is smaller. In the limiting case of no uncertainty, the solution jumps from partial compliance to full noncompliance to a zero standard, with no possible violations of positive standards.

# 5 The Optimal Separating Policy

We now turn to the case where the regulator sets a policy contingent on type. In this case, the regulator maximizes social welfare given by (5), considering the firm's optimal behavior and the incentive compatibility constraints given by (4). We first present a useful preliminary result:

**Proposition 6** *Let $(s_1^*, s_2^*, p_1^*, p_2^*)$ be the optimal separating policy. Then, $p_1^* \leq p^c(s_1^*, \theta_1)$.*

As in the pooling case, the optimal policy does not induce full compliance. The regulator now solves the following problem:

$$Max_{s_1,s_2,p_1,p_2} \{\gamma_1 (\theta_1 b(s_1) - d(s_1) - p_1 c) + \gamma_2 (\theta_2 b(n_2) - d(n_2) - p_2 c)\}$$

$$\begin{aligned} \text{s.t.} \quad & p_1 \leq p^c(s_1, \theta_1) \\ & P(s_i, p_i, \theta_i) \geq P(s_j, p_j, \theta_i) \\ & s_i \geq 0 \end{aligned} \tag{13}$$

**Proposition 7** *The optimal separating policy is given by the following condi-*

*tions:*

$$
\begin{aligned}
&\frac{\gamma_1\left(\theta_1 b'\left(n_1\right)-d'\left(n_1\right)\right) n_{1s}+\lambda \frac{\partial p^c\left(\theta_1\right)}{\partial s_1}}{\gamma_1\left(\left(\theta_1 b'\left(n_1\right)-d'\left(n_1\right)\right) n_{1p}-c\right)-\lambda}=\frac{\frac{\partial P\left(s_1^*, p_1^*, \theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_1^*, p_1^*, \theta_2\right)}{\partial p_1}} \\
&\frac{\gamma_1\left(\theta_1 b'\left(n_1\right)-d'\left(n_1\right)\right) n_{1s}+\lambda \frac{\partial p^c\left(\theta_1\right)}{\partial s_1}}{\gamma_2\left(\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right) n_{2p}-c\right)}=-\frac{\frac{\partial P\left(s_1^*, p_1^*, \theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_2^*, p_2^*, \theta_2\right)}{\partial p_2}} \\
&\frac{\gamma_1\left(\theta_1 b'\left(n_1\right)-d'\left(n_1\right)\right) n_{1s}+\lambda \frac{\partial p^c\left(\theta_1\right)}{\partial s_1}}{\gamma_2\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right) n_{2s}}=-\frac{\frac{\partial P\left(s_1^*, p_1^*, \theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_2^*, p_2^*, \theta_2\right)}{\partial s_2}} \\
&s_2^* \geq 0,\ \eta_2 \geq 0,\ s_2^* \eta_2=0 \\
&\lambda \geq 0,\ p_1^* \leq p^c\left(s_1^*, \theta_1\right),\ \lambda\left(p_1^*-p^c\left(s_1^*, \theta_1\right)\right)=0 \\
&P\left(s_2^*, p_2^*, \theta_2\right)=P\left(s_1^*, p_1^*, \theta_2\right)
\end{aligned} \tag{14}
$$

*where $(\lambda, \mu_i, \eta_i)$ are the Lagrange multipliers associated with problem (13) and $n_i$ is type $\theta_i$'s optimal response to $(s^*, p^*)$ given by* (6).

If the policy is separating, (14) implies $s_1^* > s_2^* \geq 0$ and $p_1^* > p_2^*$. Thus, type $\theta_1$ faces both a larger standard and a larger probability of inspection in order to preserve incentive compatibility. Here, the standard for type $\theta_2$ could be zero but not necessarily, since $\eta_2 \geq 0$. By (14), type $\theta_2$ is indifferent between $(s_1^*, p_1^*)$ and $(s_2^*, p_2^*)$. By Lemma 2, this means that type $\theta_1$ strictly prefers $(s_1^*, p_1^*)$. At the optimal separating policy, type $\theta_1$ is over-enforced and type $\theta_2$ is under-enforced with respect to the complete information case. type $\theta_2$ is under-enforced and type $\theta_1$ is over-enforced with respect to the complete information case. If the regulator were to naively impose the complete information solution, type $\theta_2$ would find it profitable to misreport its type, and this is why type $\theta_2$'s incentive compatibility constraint is binding.

The optimality conditions mean that, at the optimum, $(s_1, s_2, p_1, p_2)$ are such that the marginal rate of substitution between each pair of variables in terms of efficiency of the induced pollution levels equals the marginal rate of substitution between that pair of variables to induce type $\theta_2$'s truthful revelation.

Observe that the optimal separating policy can induce either partial compliance or full noncompliance (see Figure 6 for an illustration of the first case). In any event, the standard for type $\theta_2$ need not be zero. In this case, type $\theta_2$ is under-enforced with respect to the complete information case. Since the slope of $\theta_2$'s indifference curve is larger than the slope of the curve where $n_2$ is constant,[19] moving along the indifference curve towards $s = 0$ means that $n_2$ increases, so the under-enforcement problem is worse. Similarly to the pooling case, the lower the standard, the lower the enforcement costs. Now there is a trade-off between these enforcement costs and the under-enforcement problem of type $\theta_2$. Thus, the smaller (larger) the monitoring costs, the more (less) likely the optimal standard for type $\theta_2$ is positive. By contrast with the pooling case, the larger (lower) $\theta_1$'s profitability, the larger (lower) type $\theta_2$'s under-enforcement problem associated with $s = 0$. Therefore, it is more likely to have $s_2 > 0$ ($s_2 = 0$) when $\theta_1$ is large (small).

In the next example, we illustrate all these results for the separating case.

[19] See footnote 22.

## 5.1 Example 2

We consider the same functions of Example 1. By (8), type $\theta_2$'s expected profits are:

$$P(s,p,1)=\frac{1-2p(1-2s)+p^2}{4p} \tag{15}$$

since type $\theta_2$ violates the standard.

Therefore, the incentive compatibility constraint of Proposition 7 reads:

$$\frac{1-2p_1(1-2s_1)+p_1^2}{4p_1}=\frac{1-2p_2(1-2s_2)+p_2^2}{4p_2} \tag{16}$$

From (15), we have $\frac{\partial P(s_i,p_i,\theta_2)}{\partial s_i}=1$ and $\frac{\partial P(s_i,p_i,\theta_2)}{\partial p_2}=-\frac{1-p^2}{4p^2}$. The following equations and (16) characterize the optimal separating policy:

$$\begin{aligned}
&\frac{\gamma_1(p_1(\theta_1+1-2s_1)-\theta_1)}{\gamma_1\left(\theta_1 p_1(\theta_1+1-2s_1)-\theta_1^2+2cp_1^3\right)+2\lambda p_1^3}=\frac{2}{1-p_1^2}\\
&\left(\frac{\gamma_1}{\gamma_1-1}\right)\left(\frac{p_2}{p_1}\right)\left(\frac{p_1(\theta_1+1-2s_1)-\theta_1}{2p_2(1-s_2)-1+2cp_2^3}\right)=\frac{2}{1-p_2^2}\\
&\gamma_1\frac{p_1(\theta_1+1-2s_1)-\theta_1}{p_1}=(\gamma_1-1)\frac{2p_2(1-s_2)-1}{p_2}-\eta_2\\
&s_2\geq 0,\ \eta_2\geq 0,\ \eta_2 s_2=0\\
&\lambda\geq 0,\ p_1\leq\theta_1,\ \lambda(p_1-\theta_1)=0
\end{aligned}$$

We have computed the results for different values of the parameters. While we can find solutions that induce partial compliance, however we cannot find a solution which induces full noncompliance for any feasible values of the parameters. Therefore, if the policy were to be separating, in this case it would induce partial compliance. Thus, $p_1=\theta_1$.

In Figure 7, we present the relationship between the optimal standards and the monitoring costs, for different values of $\theta_1$ and $\gamma_1 = 0.5$, that is, when there is large uncertainty. If $\theta_1 = 0.5$, there does not exist a separating solution when $c \in [0, 0.3125]$. While type $\theta_1$ always complies with $s_1$, type $\theta_2$ violates a strictly positive standard $s_2$ when $c \in [0.3125, 0.58853]$. Finally, $s_2 = 0$ if $c > 0.58853$. Note that both types' standards decrease when monitoring costs increase. Also, both inspection probabilities decrease when monitoring costs increase. When $\theta_1 = 0.8$, we observe the same pattern, but here, the interval where type $\theta_2$ violates a strictly positive standard is larger, i,e., $c \in [0.016, 0.70177]$. Therefore, it is more likely that we find noncompliance to strictly positive standards when $\theta_1$ is large, since type $\theta_2$'s under-enforcement problem associated with $s = 0$ is worse in this case.

We find an analogous structure of the solution under different values of $\gamma_1$. However, it is interesting to see that, the smaller $\gamma_1$, the smaller the intervals of the monitoring costs where type $\theta_2$ violates a positive standard. Thus, if $\gamma_1 = 0.1$, we find that type $\theta_2$ violates a positive standard when $c \in [0.0625, 0.069524]$ if $\theta_1 = 0.5$ and when $c \in [0.0032, 0.0804]$ if $\theta_1 = 0.8$. Conversely, if $\gamma_1 = 0.9$, these intervals are, respectively, $c \in [0.5625, 2.304]$ and $c \in [0.028828, 2.8518]$.

Finally, comparing Figures 5 and 7, the interval of the monitoring costs for which we obtain violations to strictly positive standards under the pooling policy always contains lower values than the interval under the separating policy. Regarding social welfare, we have made some computations which show that a pooling policy may be preferred to a separating policy. For example, if $c = 0.58853$, $\theta_1 = 0.5$ and $\gamma_1 = 0.5$, we obtain $sw(pool) = -0.11822 > sw(sep) = -0.1478$. Alternatively, if $c = 0.069524$, $\theta_1 = 0.5$ and $\gamma_1 = 0.1$, we have $sw(pool) = 0.191169 > sw(sep) = 0.1909$. Therefore, this example suggests that separating policies may not always be preferred to pooling policies.

## 6 Conclusions

In this paper, we have studied optimal regulatory policies composed of pollution standards, probabilities of inspection and fines for noncompliance in a context of asymmetric information and imperfect enforcement, an approach different from that which has been done in the literature. Our model is able to explain a salient feature of environmental regulation, namely violations to strictly positive standards, a result that is not possible under either complete information or incomplete information subject to perfect enforcement, the two approaches studied until now within the principal-agent approach.

We have shown that violations to positive standards are more likely when monitoring costs are low. Since a positive standard means that the fine for noncompliance is not maximum, this result is more likely when enforcement costs are less important than the costs associated with clean type's over-enforcement or dirty type's under-enforcement, depending on the policy being pooling or separating, respectively. Regarding clean type's profitability, we have seen that positive standards are more likely under intermediate or large clean type's profitability, depending again on the policy being pooling or separating, respectively. Finally, regulator's uncertainty also matters. On one hand, the larger the uncertainty, the more likely we obtain a positive standard in the pooling case. On the other hand, the larger the likelihood of the clean type, the more likely a positive standard in the separating case.

Our results have three implications on the previous literature. First, we can rationalize violations to positive standards. This suggests that restricting attention to incentive compatible environmental taxation (where all the pollution levels are punishable) may be restrictive. Second, we have shown that the optimal policy never induces full compliance. Therefore, concentrating on the subset of perfectly enforceable policies may be also restrictive. Finally, some computations have shown that separating policies may not always be socially preferred to pooling policies. This implies that, under some circumstances, information collection may be useless.

There would be no substantial change if we considered a continuum of types instead of the two types model presented here. If the optimal policy induced some types to comply and others to violate the standards, the latter ones would be the dirtiest. In that case, the optimal policy would imply partial pooling: the compliant types would be confronted to the same policy to avoid misreporting.

However, although we have proven that violations to positive standards are possible under both the pooling and the separating schemes, we do not have a definite answer to the question of under what conditions pooling could be socially preferred to separating. This point needs more investigation.

# 7 Appendix

**Proof of Lemma 1.**

When $P(s,p,\theta_i) = \theta_i b(s)$, the function is strictly increasing and concave in $s$, but it does not depend on $p$. Also, $\theta_2 b(s) > \theta_1 b(s)$. Conversely, when

$P(s,p,\theta_i) = \pi(s,p,\theta_i)$, we have:

$$\pi_s(s,p,\theta_i) = pF'(n_i - s) \geq 0 \tag{17}$$

$$\pi_{ss}(s,p,\theta_i) = pF''(n_i - s)(n_{is} - 1) \leq 0 \tag{18}$$

$$\pi_p(s,p,\theta_i) = -F(n_i - s) < 0 \tag{19}$$

$$\pi_{pp}(s,p,\theta_i) = -F'(n_i - s)\, n_{ip} > 0 \tag{20}$$

$$\pi_{sp}(s,p,\theta_i) = F'(n_i - s) + pF''(n_i - s)\, n_{ip} > 0 \tag{21}$$

where $n_i = n(s,p,\theta_i)$. Also, we trivially obtain that $\pi(s,p,\theta_2) > \pi(s,p,\theta_1)$. Summing up both possibilities we obtain the desired result.

Finally, $\pi(s,0,\theta_i) = \max_{e>0} \theta_i b(e) = \theta_i b(\widetilde{e})$, for all $i$. Thus, $P(s,0,\theta_i) = \theta_i b(\widetilde{e})$, as desired. ■

**Proof of Lemma 2.**

In $\theta_i$'s compliance region, the expected payoff is $\theta_i b(s)$, that is, it does not depend on the probability of inspection. Therefore, indifference curves have an infinite slope. In the noncompliance region, the expected payoff is $\pi(s,p,\theta_i) = b(n(s,p,\theta_i)) - pF(n(s,p,\theta_i) - s)$. Implicitly differentiating $\pi(s,p,\theta_i) = k$, we obtain:

$$\frac{dp}{ds}\,|_{\pi=k} = \frac{pF'(n(s,p,\theta_i) - s)}{F(n(s,p,\theta_i) - s)} > 0 \tag{22}$$

Now, differentiating (22) with respect to $s$ we have:

$$\frac{d^2p}{ds^2}\,|_{\pi=k} = \frac{F'}{F}\frac{dp}{ds}\,|_{\pi=k} + \frac{p}{F^2}\left(F''F - (F')^2\right)(n_{is} - 1) > 0 \tag{23}$$

since $n_{is} < 1$ and $F''F - (F')^2 < 0$.

(For analytical convenience, we prove the last part considering a continuum of types. The result is easily adapted to the case in which $\theta$ takes discrete values.)

In the compliance region, both types' indifference curves are vertical. In the partial compliance region, $\theta_1$'s are vertical and $\theta_2$'s are strictly increasing In the full noncompliance region, we differentiate (22) with respect to $\theta$ to obtain:

$$\frac{d^2p}{dsd\theta} \mid_{\pi=k} = \frac{F''F - (F')^2}{(F)^2} pn_\theta (s, p, \theta) \tag{24}$$

Since $F''F - (F')^2 < 0$, $\frac{d^2p}{dsd\theta} \mid_{\pi=k}$ and $n_\theta (s, p, \theta)$ have the opposite sign. Differentiating (6) with respect to $\theta$, we obtain $n_\theta (s, p, \theta) = -\frac{b'}{\theta b'' - pF''} > 0$. Therefore, $\frac{d^2p}{dsd\theta} \mid_{\pi=k} < 0$, as desired. ■

**Proof of Proposition 3.**

Assume first that $p \geq p^c (s, \theta_2)$, that is, the pooling policy induces full compliance. The problem the regulator faces in this case is:

$$\begin{aligned} &Max_{s,p} \sum_{i=1}^{2} \gamma_i \left[\theta_i b \left(e \left(s, p, \theta_i\right)\right) - d \left(e \left(s, p, \theta_i\right)\right)\right] - cp \\ &s.t.\ p \geq p^c (s, \theta_2) \end{aligned} \tag{25}$$

Since $p \geq p^c (s, \theta_2)$, we then have $e (s, p, \theta_i) = s$ for all $i$. The Lagrangian of problem (25) is the following:

$$L (s, p, \lambda) = \sum_{i=1}^{2} \gamma_i \left[\theta_i b (s) - d (s)\right] - cp - \lambda \left(p^c (s, \theta_2) - p\right)$$

where $\lambda \geq 0$ is the corresponding Lagrange multiplier. The interior solution is given by the following Kuhn-Tucker conditions:[20]

$$\gamma_1 \left(\theta_1 b'(s) - d'(s)\right) + \gamma_2 \left(\theta_2 b'(s) - d'(s)\right) - \lambda \frac{dp^c(s,\theta_2)}{ds} = 0$$

$$c - \lambda = 0$$

$$\lambda \left(p^c(s,\theta_2) - p\right) = 0$$

Since $c > 0$, we have $\lambda > 0$ and $p^c(s,\theta_2) = p$, which leads to:

$$\begin{aligned} &\sum_{i=1}^{2} \gamma_i \left(\theta_i b'(s^*) - d'(s^*) - c\frac{dp^c(s^*,\theta_2)}{ds}\right) = 0 \\ &p^* = p^c(s^*,\theta_2) \end{aligned} \tag{26}$$

Since $\theta_1 < \theta_2$, (26) implies that $\theta_1 b'(s^*) - d'(s^*) - c\frac{dp^c(s,\theta_2)}{ds} < 0$ and $\theta_2 b'(s^*) - d'(s^*) - c\frac{dp^c(s,\theta_2)}{ds} > 0$. This last expression can be written as $\left(\theta_2 b'(s^*) - d'(s^*)\right)\frac{ds}{dp^c} - c < 0$, since $\frac{dp^c(s,\theta_2)}{ds} < 0$. By the continuity of the sanction at $e - s = 0$, we can infinitesimally decrease $p$ to increase social welfare, without affecting $\theta_1$'s behavior. Therefore, a pooling policy which induces full compliance is never optimal.

We now consider the case in which $p^c(s,\theta_1) < p \leq p^c(s,\theta_2)$. This corresponds to the partial compliance region, where the clean type strictly prefers to comply with the standard. Now, the problem is:

$$\begin{aligned} Max_{s,p} \quad & \sum_{i=1}^{2} \gamma_i \left[\theta_i b\left(e(s,p,\theta_i)\right) - d\left(e(s,p,\theta_i)\right)\right] - cp \\ s.t. \quad & p > p^c(s,\theta_1) \end{aligned} \tag{27}$$

[20] The assumptions of the model ensure that these conditions are necessary an sufficient for an interior optimum. This continues to hold for the remaining optimality results of the paper.

where $s = e(s, p, \theta_1)$ and $n_2 = e(s, p, \theta_2)$. The Kuhn-Tucker conditions are the following:

$$\gamma_1 (\theta_1 b'(s) - d'(s)) + \gamma_2 (\theta_2 b'(n_2) - d'(n_2)) n_{2s} - \lambda \frac{dp^c(s, \theta_1)}{ds} = 0$$

$$\gamma_2 (\theta_2 b'(n_2) - d'(n_2)) n_{2p} - c + \lambda = 0$$

$$\lambda (p^c(s, \theta_1) - p) = 0$$

where $\lambda \geq 0$ is the corresponding Lagrange multiplier of problem $(27)$.

Observe that $\lambda = c - \gamma_2 (\theta_2 b'(n_2) - d'(n_2)) n_{2p} \geq 0$. Since $p > p^c(s, \theta_1)$, $\lambda$ must be equal to 0. This implies that $c = \gamma_2 (\theta_2 b'(n_2) - d'(n_2)) n_{2p}$, which means that $c > \left(\theta^2 b'\left(n^2\right) - d'\left(n^2\right)\right) n_{2p}$, since $\gamma_2 < 1$. Therefore, welfare can increase if $p$ decreases infinitesimally, since type $\theta_1$ continues to comply with $s$. Therefore, the optimal pooling policy cannot be such that $p > p^c(s, \theta_1)$. ■

**Proof of Corollary 5.**

Assume, to the contrary, that $\lambda = 0$, that is, the optimal policy induces full noncompliance when $c = 0$. By (11), we have $\gamma_1 A_1 n_{1p} = -\gamma_2 A_2 n_{2p}$, where $A_i = \theta_i b'(n_i) - d'(n_i)$ for all $i$. Considering (12), it must be true that:

$$\eta = -\gamma_2 A_2 \left( \frac{n_{2p}}{n_{1p}} n_{1s} - n_{2s} \right)$$

Since $A_2 < 0$ and $\frac{n_{2p}}{n_{1p}} n_{1s} - n_{2s} > 0$ (see footnote 17), we then have $\eta < 0$, which contradicts the fact that the solution induces full noncompliance.

**Proof of Proposition 6.**

First, a separating policy in the full compliance region is not possible since indifference curves do not cross. Thus, any attempt to set a separating policy would induce misreporting of the type with the lowest assigned standard. Also, any policy that assigns the same standard but a different probability is incentive compatible but suboptimal, since both probabilities can be decreased till the boundary $p^c(s, \theta_2)$ without distorting incentives and reducing monitoring costs.

Next, consider the case of partial compliance where $p_1 > p^c(s_1, \theta_1)$. The problem is:

$$
\begin{aligned}
& Max_{s_1,s_2,p_1,p_2} \{\gamma_1 (\theta_1 b(s_1) - d(s_1) - p_1 c) + \gamma_2 (\theta_2 b(n_2) - d(n_2) - p_2 c)\} \\
\text{s.t.} \quad & p_1 > p^c(s_1, \theta_1) \\
& P(s_i, p_i, \theta_i) \geq P(s_j, p_j, \theta_i), \ i, j = 1, 2, \ i \neq j \\
& s_i \geq 0, \ i = 1, 2
\end{aligned} \tag{28}
$$

Considering $\lambda \geq 0$ to be the Lagrange multiplier associated with the first restriction in problem (28), the Kuhn-Tucker conditions associated with $p_1$ are:

$$
\begin{aligned}
& \lambda = \gamma_1 c \\
& \lambda \geq 0, \ \lambda (p^c(s_1, \theta_1) - p_1) = 0
\end{aligned}
$$

Since $p_1 > p^c(s_1, \theta_1)$, we then must have $\lambda = 0$. However, $\lambda = \gamma_1 c > 0$, which is a contradiction. Therefore, $p_1 \leq p^c(s_1, \theta_1)$, as desired. ■

**Proof of Proposition 7.**

The Kuhn-Tucker conditions of problem (13) are the following:

$$\gamma_1 \left( \left( \theta_1 b' \left( n_1 \right) - d' \left( n_1 \right) \right) n_{1p} - c \right) - \lambda + \mu_1 \frac{\partial P \left( s_1^*, p_1^*, \theta_1 \right)}{\partial p_1} - \mu_2 \frac{\partial P \left( s_1^*, p_1^*, \theta_2 \right)}{\partial p_1} = 0$$

$$\gamma_1 \left( \theta_1 b' \left( n_1 \right) - d' \left( n_1 \right) \right) n_{1s} + \lambda \frac{\partial p^c \left( \theta_1 \right)}{\partial s_1} + \mu_1 \frac{\partial P \left( s_1^*, p_1^*, \theta_1 \right)}{\partial s_1} - \mu_2 \frac{\partial P \left( s_1^*, p_1^*, \theta_2 \right)}{\partial s_1} = 0$$

$$\gamma_2 \left( \left( \theta_2 b' \left( n_2 \right) - d' \left( n_2 \right) \right) n_{2p} - c \right) - \mu_1 \frac{\partial P \left( s_2^*, p_2^*, \theta_1 \right)}{\partial p_2} + \mu_2 \frac{\partial P \left( s_2^*, p_2^*, \theta_2 \right)}{\partial p_2} = 0$$

$$\gamma_2 \left( \theta_2 b' \left( n_2 \right) - d' \left( n_2 \right) \right) n_{2s} - \mu_1 \frac{\partial P \left( s_2^*, p_2^*, \theta_1 \right)}{\partial s_2} + \mu_2 \frac{\partial P \left( s_2^*, p_2^*, \theta_2 \right)}{\partial s_2} = 0$$

$$s_1^* > 0, \; \eta_1 = 0$$

$$s_2^* \geq 0, \; \eta_2 \geq 0, \; s_2^* \eta_2 = 0$$

$$\lambda \geq 0, \; p_1^* \leq p^c \left( s_1^*, \theta_1 \right), \; \lambda \left( p_1^* - p^c \left( s_1^*, \theta_1 \right) \right) = 0$$

$$\mu_i \geq 0, \; P \left( s_i^*, p_i^*, \theta_i \right) \geq P \left( s_j^*, p_j^*, \theta_i \right), \; \mu_i \left( P \left( s_i^*, p_i^*, \theta_i \right) - P \left( s_j^*, p_j^*, \theta_i \right) \right) = 0$$

where $\lambda \geq 0$, $\mu_i \geq 0$, $\eta_i \geq 0$ are, respectively, the Lagrange multipliers associated with the restrictions in problem (13).

Assume first that $\mu_1 = \mu_2 = \eta_2 = 0$. This implies that $\theta_2 b' \left( n_2 \right) - d' \left( n_2 \right) = 0$ and $c = 0$, since $n_{2p} < 0$. Since $c > 0$, either one of the incentive compatibility constraints must be binding or $\eta_2 \geq 0$.[21] Assume first that $\mu_1 \geq 0$ and $\mu_2 = \eta_2 = 0$. However, $n_2$ can be kept constant decreasing both $(s_2, p_2)$ through expression (6) without distorting the incentive compatibility constraints.[22] Therefore, $\mu_1 \geq 0$, $\mu_2 = \eta_2 = 0$ is not possible.

Now, consider $\mu_1 = 0$, $\mu_2 = 0, \eta_2 \geq 0$. In this case, first order conditions would reduce to $\theta_1 b' \left( s_1 \right) - d' \left( s_1 \right) = c \frac{dp^c(\theta_1)}{ds_1}$ and $\left( \theta_2 b' \left( n_2 \right) - d' \left( n_2 \right) \right) n_{2p} = c$, respectively, the optimal compliance solution for type $\theta_1$ and the optimal

[21] It is easy to see that both incentive compatibility constraints cannot be binding except in the case of a pooling policy. Thus, $\mu_1 \geq 0$, $\mu_2 \geq 0$ is not possible if the policy is separating.

[22] To see this, consider (22) and $\frac{dp}{ds} \mid_{n_2} = -\frac{n_{2s}}{n_{2p}} = \frac{pF''}{F'}$ to conclude that $\frac{dp}{ds} \mid_{n_2} < \frac{dp}{ds} \mid_{P_2}$, since $\left( F' \right)^2 - FF'' > 0$. By Lemma 2, we then have $\frac{dp}{ds} \mid_{n_2} < \frac{dp}{ds} \mid_{P_1}$.

noncompliance solution for type $\theta_2$ if information were complete. But, in this case, type $\theta_2$ would prefer to misreport its type. Therefore, $\mu_1 = 0$, $\mu_2 = 0, \eta_2 \geq 0$ is not possible. For the same reason, $\mu_1 \geq 0$, $\mu_2 = 0, \eta_2 \geq 0$ is not possible either.

Therefore, $\mu_1 = 0$ and $\mu_2 > 0$. As for $\eta_2$, both $\eta_2 = 0$ and $\eta_2 \geq 0$ are compatible with the solution, thus obtaining the desired result. ■

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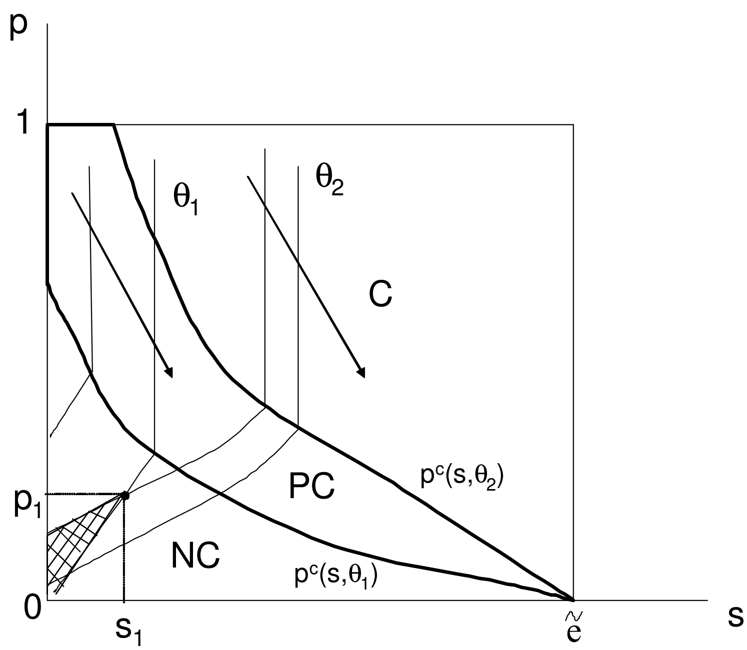


Figure 1: The compliance and noncompliance regions

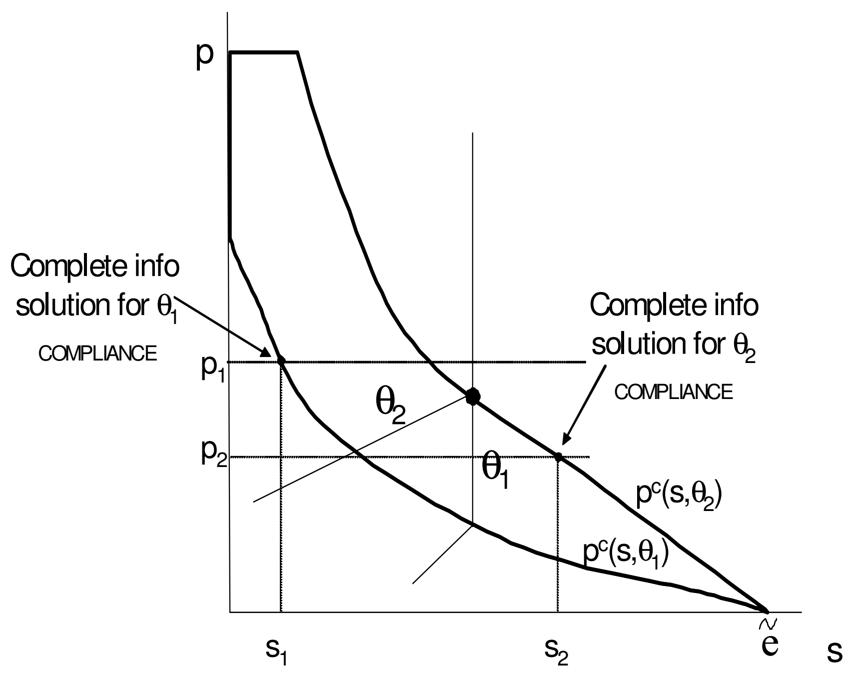


Figure 2: The pooling policy under full compliance

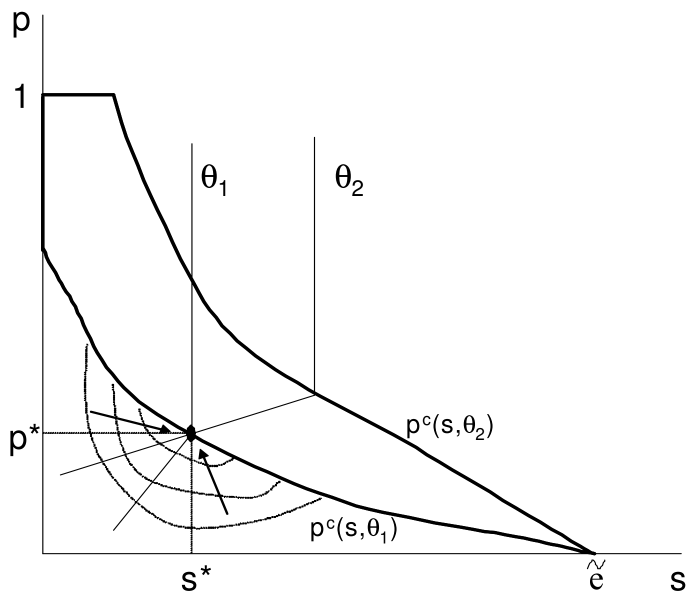


Figure 3: The optimal pooling policy induces partial compliance

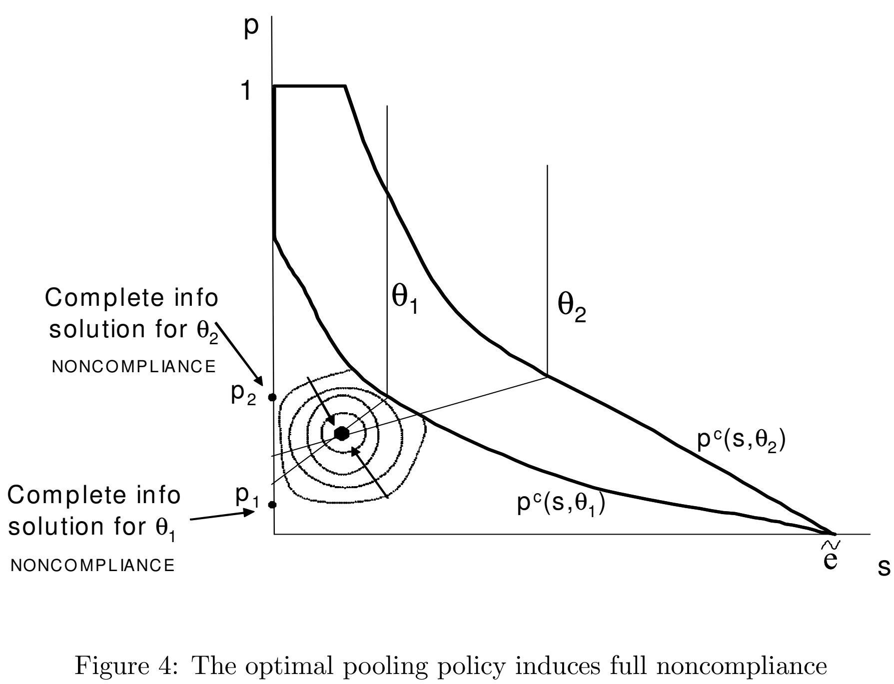


Figure 4: The optimal pooling policy induces full noncompliance

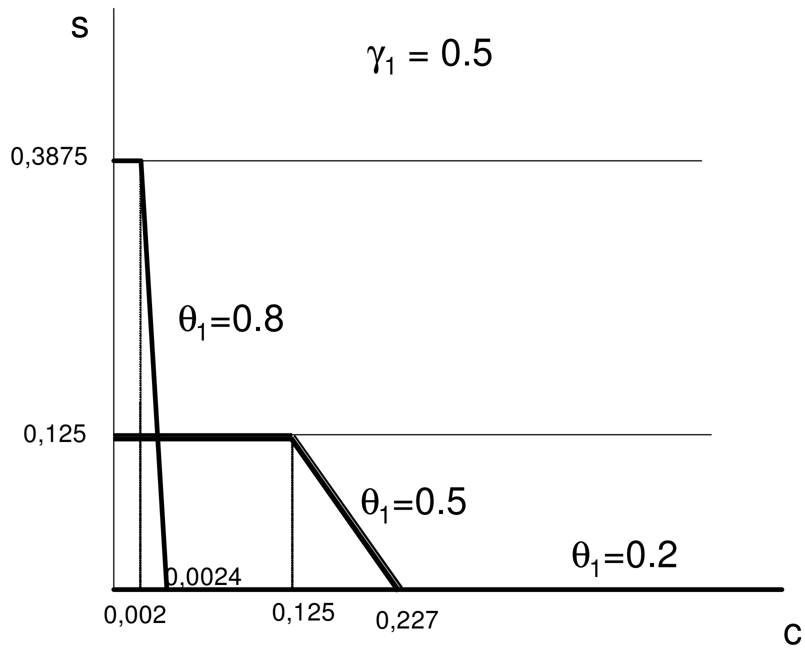


Figure 5: The optimal standard in the pooling case

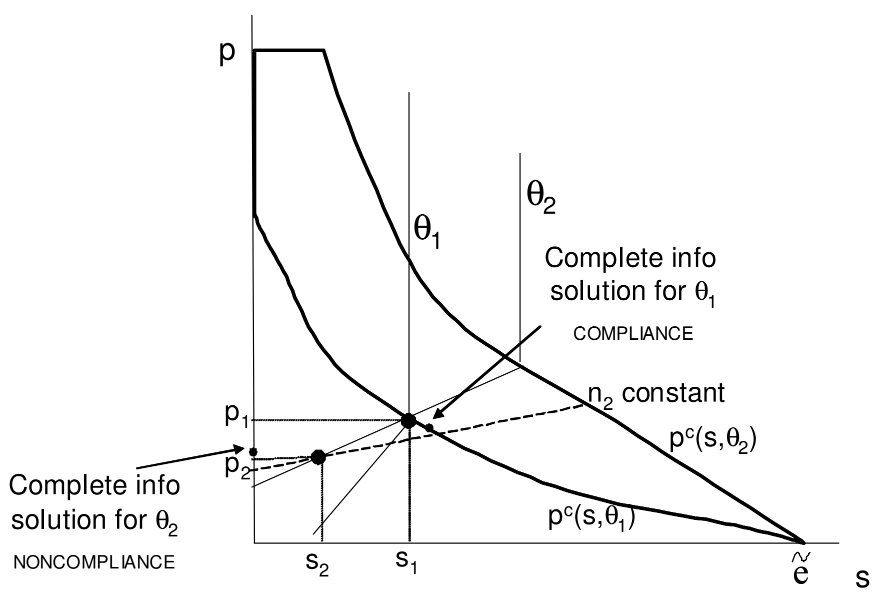


Figure 6: The optimal separating policy when it induces partial compliance

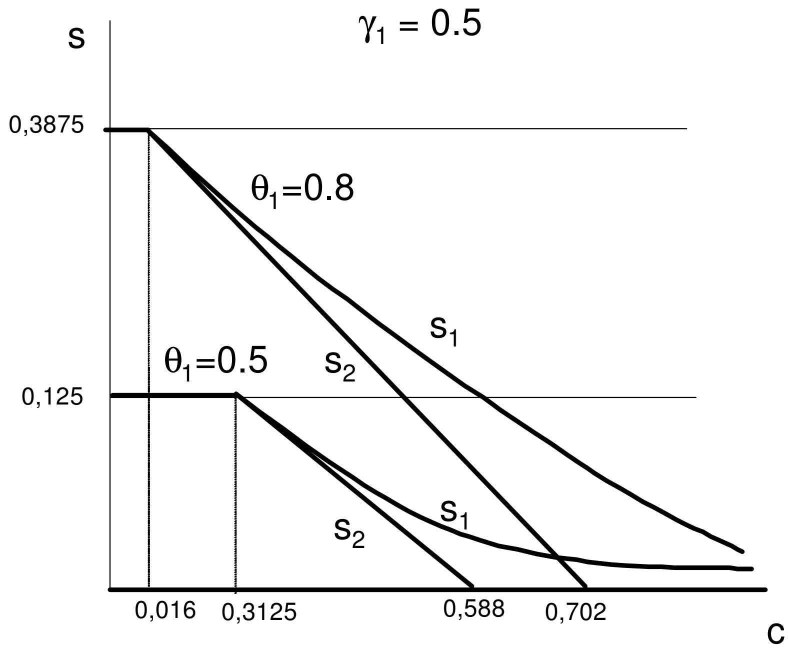


Figure 7: The optimal standards in the separating case